PYL742: **Gravitation and Cosmology**

Major Online Examination

Sep 5, 2020 3:00 PM – 6:00 PM

Instructions

1. Read the Instructions carefully
2. On the first page, at the top, write your name, your entry code, your date of birth in dd/mm/yyyy format and finally, the pincode for your location. This is important.
3. Answer all the parts of a question in one place
4. Calculations, rough work, and answer for each question must all be together and submitted. Any information that you have looked up, from books or the web must be mentimed.
5. All answers must be justified, howsoever short.
   Otherwise no credit will be given

Unless otherwise stated, it us understood that we employ natural units: $\hbar = c = G = 1$.

Question 1. (a) (10 marks) Perform the following boosts successively on the momentum four vector and give the final result: $L_Y(-V)L_X(-V)L_Y(V)L_X(V)$. In particular, find the angle between the three momenta before and after the Lorentz transformations.

Question 2. (a) (10 marks) A static charge is distributed uniformly in S over a sphere of radius R ina framseS_1. Find The electric and magnetic fields which are observed in the frame S_2 moving along the X direction with a speed V with respect to S_2. V is the largest integer in your entry code divided by 9. If the integer happens to be 9, take it to be the next largest integer.

(b) (5 marks) You are given $\vec{E} = E_0\hat{i}$ and $\vec{B} = \frac{E_0}{n}\hat{k}$ in some frame. Find the frame in which it is pure electric or pure magnetic field. n is the third integer in your pin code.

Question 3. (a) (10 marks) A satellite is in an orbit around the earth at a distance mR, where R is the radius of the Earth and m is the product of the first two smallest (nonvanishing) integers in your entry code. Compute the corrections to the rate of clock in the satellite coming from both the Doppler and gravitational effects.

(b) (5 marks) Determine the total red/blue shift if a radiation is emitted from the satellite is received by an observer o the Earth.

(c) (5 marks) At what distance will be the shift entirely due to gravitation?

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Question 4. (a) (5 marks) Determine the equation of motion for a particle in the metric given by

$$ds^2 = (1 + \alpha \exp(-2t^2))dt^2 - (\beta \tanh^2 x)dx^2$$

where α, β are the largest even numbers in your date of birth and entry code respectively.

(b) (5 marks) What is the energy momentum tensor responsible for this metric?

Question 5. This is based on the Schwarschild metric. The mass of the source is k times the solar mass, where k is the sum of all the integers in all the adta you provided (date of birth, entry code, pincode).

(a) (10 marks) Consider the spatial equatorial "plane" corresponding to $\theta = \pi/2$. Looking at the metric in this plane, deduce whether it is Euclidean or not. Characrerise its curvature explicitly.

(b) (5 marks) Plot, as a function of distance from the singularity, , the rate at which a clock runs, relative to a clock at an infinite distance. Mark important coordinates along the coordinate axes explicitly.

Question 6. (a) (15 marks) Look up the FRW metric. and set up the equation of motion for a test particle.

(b) (15 marks) Set up Maxwell's equations in this background metric. How would they differ, qualitatively, for positive and negatively curved cases?

Some useful formulas

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\begin{aligned}
D_\mu A^\nu &= \partial_\mu A^\nu + \Gamma^\nu_{\alpha\rho} A^\rho \\
D_\mu A_\nu &= \partial_\mu A_\nu - \Gamma^\alpha_{\mu\nu} A_\alpha \\
\Gamma^\mu_{\nu\rho} &= \frac{1}{2} g^{\mu\alpha} \left\{ g_{\alpha\nu|\rho} + g_{\alpha\rho|\nu} - g_{\nu\rho|\alpha} \right\} \\
R_{\mu\nu\rho\sigma} &= g_{\mu\alpha} \left\{ \Gamma^\alpha_{\rho\sigma|\nu} - \Gamma^\alpha_{\rho\nu|\sigma} + \Gamma^\alpha_{\eta\rho} \Gamma^\eta_{\nu\sigma} - \Gamma^\alpha_{\eta\sigma} \Gamma^\eta_{\nu\rho} \right\}
\end{aligned}
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