Semester II, 2021-22
MTL108: Major on 09/04/2022

Max Marks: 40 Max Time: 8:15-10:15 am (+5 minutes) including uploading time

The exam is offline. The answer scripts with your name and entry number must be uploaded on gradescope ONLY. The access of GS will be permitted only after 9:50 am. The mapping of answers on gradescope must be CORRECT. Any unfair means will force zero in major. Also, provide a complete step-wise details of each answer. Please abide by these instructions.

1. Consider a drone flying above a sea whose see shore can be assumed to be represented by a line $x + y = 20$. There is a vertical signal tower T standing on the sea shore at the position (a, b). The flying drone frequently send signals to T which are used to estimate drone's position. Let the drone's position coordinates X and Y follow $N(9, 1)$ and $N(14, 4)$, respectively. A random data of signals shared by the drone on its position is collected as (X_i, Y_i), $i = 1, \ldots, n$, all X_i and Y_i are independent random variables.

   Determine the suitable position (a, b) on the sea shore for setting up the signal tower T so that the expected value of the root mean square distance over the observed n positions of the drone from the position of T equals 10. [7]

2. Suppose $X_1, \ldots, X_{20},\ Y_1, \ldots, Y_{20},\ Z_1, \ldots Z_{10} \sim N(0, 0.5)$, be independent random samples. Let $A = \dfrac{\sum_{i=1}^{20}(X_i - Y_i)}{2\sqrt{\sum_{j=1}^{10} Z_j^2}}$ and $B = 2(19S_X^2 + 9S_Z^2)$, where S_X^2 is the sample variance of X_i's and S_Z^2 is the sample variance of Z_j's.

   Find the probability $P(5 < A^2 < 10,\ 20 < B < 30)$. [7]

3. For the 15-year period between 2005 and 2020, the average annual return of two financial assets A and B be 25% and 22% and their standard deviations are 19% and 18% respectively. A fund manager of some mutual fund house feels that the returns from A is better than the returns from B assuming that the two assets' historical annual returns populations posses the same variance. To confirm this, he first perform a test to determine if his assumption on the variance of populations holds merit, and subsequently he test the out-performance of annual returns of A over annual returns of B, both tests are tested at 10% level of significance. Set up the hypotheses for tests. What is the conclusion at the end of the two tests? [6]

4. A manufacturer set up a new technology of producing CPVC pipes. He feels that this new technique will reduce the proportion of defects in pipes. To verify this, a random sample of 420 pipes produced by new technology is picked. And a random sample of 460 pipes produced by the existing technique is chosen. Out of this 24 (new) and 42 (existing) are found having minor defects. Set the null hypothesis and test it at 5% level of significance. What is the p-value of the test? Based on this p-value, what is an appropriate conclusion of the test. [5]

5. Let a random variable X follows the pdf $f(x) = \frac{2x}{\lambda^2} \exp\left(-\left(\frac{x}{\lambda}\right)^2\right)$, $x > 0$, $\lambda > 0$.
A random data is observed from it as follows: 11.0, 15.2, 18.4, 21.5, 25.2.
Use this data to find the MLE of λ. [5]

6. A company has 1000 employees all insured with a policy that provides death benefit of Rs 30000 each to a family member of an employee who expires while in service. There is a 1.1% chance that any one employee of the company will die next year, independent of all other employees. The company establishes a fund such that the probability is at least 0.99 that the fund will cover next year's death benefits for all those employees who expire. Use the Binomial continuous correction factor to compute the minimum amount of corpus that the company must put in the fund . [5]

7. Let (X, Y) be uniformly distributed random variables on the semi-circle $\{(x, y) : x^2 + y^2 \leq 1,\ x \geq 0\}$. Write the joint pdf of (X, Y). Compute the conditional expectation $E(X\cos(\pi Y) \mid Y = \frac{1}{4})$. [5]